Journal of Novel Applied Sciences
Available online at www.jnasci.org
©2014 JNAS Journal-2014-3-S2/1682-1687
ISSN 2322-5149 ©2014 JNAS

Some irreducible subgroups of the group GL(6,2)

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ABSTRACT: In this paper we getting Some irreducible subgroups of the group $GL(6,p^k)$.

Keywords: irreducible, generating set, primitive, soluble.

INTRODUCTION

Jordan(1871a) essentially gives a table containing the numbers of conjugacy classes of maximal irreducible soluble subgroups of $GL(n,p)$ for $p^n < 10^6$. He claims there are five such classes in $GL(4,3)$, but there are only four' (see to chapter 6). This error is likely to lead to errors for larger degrees . Also' the second and third entries in the last row of this table should be swapped.

Dickson(1901' chapter 12' pp. 260-287) determine all subgroup of $PSp(2,p^k)$ (for a determination in modern terminology' see Huppert(1967' chapter 2' section 8' pp.191-214) and in (1904) he determines all Subgroups of $PSp(4,3)$.

Mitchell(1914) determines the maximal subgroups of $PSp(4,p^k)$ for odd p. Liskovec(1973) classifies the maximal irreducible $\{p,q\}$-subgroups of $GL(r^2,p)$, where q and r are primes and q is odd.Conlon(1977) determines the non–abelian q-subgroups (q prime) of $GL(q,p^k)$ and the non abelian 2-subgroups of $Sp(2,p^k)$. Harada and Yamaki (1979) count the irreducible subgroups of $GL(n,2)$ for $n \le 6$. They do not describe their methods' and the only groups they list are he insoluble ones for $n = 6$.
Their count for the soluble groups of these degree is correct .

Kondrat'ev (1985' 1986a' 1986b and 1987) determines the irreducible subgroups of $GL(7,2)$, the insoluble irreducible subgroups of $GL(8,2)$ and $GL(9,2)$, and the insoluble primitive subgroups of $GL(10,2)$.Now in this paper we getting a Some irreducible subgroups of the group $GL(6,p^k)$.

2.Pereleminares:

In this section we determine JS-maximals of $GL(6,p^k)$, for p^k =1' …' 40 and …
And we get a complete and irredundant set of conjugacy class representatives of the irreducible soluble subgroups for some p^k. For this work the first we determine the JS-maximals of $GL(2,p^k)$ and $GL(3,p^k)$ and the end we obtaind the tables of $GL(6,p^k)$, for $p^k = 1,...,40$.

2.1–The JS-maximals of $\mathrm{GL}(6,p^k)$.

Recall that we number the JS-maximales of $GL(2,p^k)$ as follows.

$M_1(2,p^k) := GL(1,p^k) \ \ wr \ \ S_2, \qquad p^k \neq 2,$

$M_2(2,p^k) := C_{p^{2k}-1} \succ C_2,$

$M_3(2,p^k) := (C_{p^k-1} \ \ \mathrm{Y} \ \ Q_8) \ \ \mathrm{N} \ \ O^-(2,2), p^k \equiv 3(\mathrm{mod}4),$

$M_4(2,p^k) := (C_{p^k-1} \ \ \mathrm{Y} \ \ Q_8) \ \ \mathrm{N} \ \ Sp(2,2), p^k \equiv 1(\mathrm{mod}4).$

And also recall that we number the JS-maximals of $GL(3,p^k)$ as follows:

$M_1(3,p^k) := GL(1,p^k) \ \ wr \ \ S_3, \qquad p^k \neq 2,$

$M_2(3,p^k) := C_{p^{3k}-1} \succ C_3,$

$M_3(3,p^k) := (C_{p^k-1} \ \ \mathrm{Y} \ \ E_{27}) \ \ \mathrm{N} \ \ Sp(2,3), p^k \equiv 1(\mathrm{mod}3).$

Therefore the JS-imprtitives of $GL(6,p^k)$ are listed below.

$M_1(6,p^k) := GL(1,p^k) \ \ wr \ \ (S_2 \ \ wr \ \ S_3), p^k \neq 2,$

$M_2(6,p^k) := GL(1,p^k) \ \ wr \ \ (S_3 \ \ wr \ \ S_2), p^k \neq 2,$

$M_3(6,p^k) := M_2(2,p^k) \ \ wr \ \ S_3,$

$M_4(6,p^k) := M_3(2,p^k) \ \ wr \ \ S_3, \qquad p^k \equiv 3(\mathrm{mod}4),$

$M_5(6,p^k) := M_4(2,p^k) \ \ wr \ \ S_3, \qquad p^k \equiv 1(\mathrm{mod}4),$

$M_6(6,p^k) := M_2(3,p^k) \ \ wr \ \ S_2,$

$M_7(6,p^k) := M_3(3,p^k) \ \ wr \ \ S_2, \qquad p^k \equiv 1(\mathrm{mod}3).$

By use from theorems 2.3 and 2.4 follows that every transitive maximal soluble subgroup of S_6 is conjugate to $S_2 \ \ wr \ \ S_3 \ \ or \ \ S_3 \ \ wr \ \ S_2.$

Also the JS-primitives of $GL(6,p^k)$ are listed below.

$M_8(6,p^k) := C_{p^{6k}-1} \times C_6 \qquad ,$

$M_9(6,p^k) := M_3(2,p^{3k}) \times C_3 \qquad , p^{3k} \equiv 3(\mathrm{mod}4),$

$M_{10}(6,p^k) := M_4(2,p^{3k}) \times C_3 \qquad , p^{3k} \equiv 1(\mathrm{mod}4),$

$M_{11}(6,p^k) := M_3(3,p^{2k}) \times C_2 \qquad , p^{2k} \equiv 1(\mathrm{mod}3),$

$M_{12}(6,p^k) := M_3(2,p^k) \otimes M_3(3,p^k), p^k \equiv 7(\mathrm{mod}12),$

$M_{13}(6,p^k) := M_4(2,p^k) \otimes M_3(3,p^k), p^k \equiv 1(\mathrm{mod}12).$

Therefore by use of JS-maximals of $GL(6,p^k)$, The table of $GL(6,p^k)$, For $p^k = 1,...,40.$ as will follows.

GL(6’1)	M_1	M_2	M_3		M_5	M_6	M_7	M_8		M_{10}	M_{11}		M_{13}
GL(6’2)			M_3			M_6		M_8			M_{11}		
GL(6’3)	M_1	M_2	M_3	M_4		M_6		M_8	M_9				
GL(6’4)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		
GL(6’5)	M_1	M_2	M_3		M_5	M_6		M_8		M_{10}	M_{11}		
GL(6’6)	M_1	M_2	M_3			M_6		M_8					
GL(6’7)	M_1	M_2	M_3	M_4		M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	
GL(6’8)	M_1	M_2	M_3			M_6		M_8			M_{11}		
GL(6’9)	M_1	M_2	M_3		M_5	M_6		M_8		M_{10}			
GL(6’10)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		
GL(6’11)	M_1	M_2	M_3	M_4		M_6		M_8	M_9	M_{10}	M_{11}		
GL(6’12)	M_1	M_2	M_3			M_6		M_8					
GL(6’13)	M_1	M_2	M_3		M_5	M_6	M_7	M_8		M_{10}	M_{11}		M_{13}
GL(6’14)	M_1	M_2	M_3			M_6		M_8			M_{11}		
GL(6’15)	M_1	M_2	M_3	M_4		M_6		M_8	M_9	M_{10}			
GL(6’16)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		
GL(6’17)	M_1	M_2	M_3		M_5	M_6		M_8		M_{10}	M_{11}		
GL(6’18)	M_1	M_2	M_3			M_6		M_8					
GL(6’19)	M_1	M_2	M_3	M_4		M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	
GL(6’20)	M_1	M_2	M_3			M_6		M_8			M_{11}		
GL(6’21)	M_1	M_2	M_3		M_5	M_6		M_8		M_{10}			
GL(6’22)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		
GL(6’23)	M_1	M_2	M_3	M_4		M_6		M_8	M_9		M_{11}		
GL(6’24)	M_1	M_2	M_3			M_6		M_8					
GL(6’25)	M_1	M_2	M_3		M_5	M_6	M_7	M_8		M_{10}	M_{11}		M_{13}
GL(6’26)	M_1	M_2	M_3			M_6		M_8			M_{11}		
GL(6’27)	M_1	M_2	M_3	M_4		M_6		M_8	M_9				
GL(6’28)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		
GL(6’29)	M_1	M_2	M_3		M_5	M_6		M_8		M_{10}	M_{11}		
GL(6’30)	M_1	M_2	M_3			M_6		M_8					
GL(6’31)	M_1	M_2	M_3	M_4		M_6	M_7	M_8	M_9		M_{11}	M_{12}	
GL(6’32)	M_1	M_2	M_3			M_6		M_8			M_{11}		
GL(6’33)	M_1	M_2	M_3		M_5	M_6		M_8		M_{10}			
GL(6’34)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		
GL(6’35)	M_1	M_2	M_3	M_4		M_6		M_8	M_9		M_{11}		
GL(6’36)	M_1	M_2	M_3			M_6		M_8					
GL(6’37)	M_1	M_2	M_3		M_5	M_6	M_7	M_8		M_{10}	M_{11}		M_{13}
GL(6’38)	M_1	M_2	M_3			M_6		M_8			M_{11}		
GL(6’39)	M_1	M_2	M_3	M_4		M_6		M_8	M_9				
GL(6’40)	M_1	M_2	M_3			M_6	M_7	M_8			M_{11}		

By use from the above we conclude that when $p^k = 2$, then there are exactly four **Js**-maximals, namely M_3, M_6, M_8 *and* M_{11}. In this part we finding a generating set for $M_{11}(6, p^k)$ and our determined irreducible soluble subgroups of $M_3(6,2), M_6(6,2), M_8(6,2)$ and $M_{11}(6,2)$.
Therefore the irreducible subgroups and generating set of their as follows.

3. Main R esult:

In this section we determine the some of irreduclble subgroups of $GL(6,2)$, as follows.

3.1.A generating set for $M_{11}(6, p^k)$.

Let F be the field of p^k elements, let $x^2 + \mu x + \lambda$ be a primitive polynomial over F, and set

$$\bar{t} = \begin{bmatrix} 1 & 0 \\ -\mu & -1 \end{bmatrix} \; and \; \bar{z} := \begin{bmatrix} 0 & 1 \\ -\lambda & -\mu \end{bmatrix}.$$

Then $\bar{t}$ has order **2**, $\bar{z}$ has order $p^k - 1$, and $\bar{z}^{\bar{t}} = \bar{z}^{p^k}$.Let E be the field of order p^{2k} That is the linear span of the powers of $\bar{z}$.Then $\bar{t}$ acting by conjugation induces an automorphism of E of order **2** which fixes F elements-wis.

Recall that $M_{11}(6,F) = M_3(3, p^k) \succ C_2$.Let ε be a primitive cube root of unity in E, say $\bar{z}^{(p^k-1)/3}$. Then by use of generating set for $M_3(3,F)$ We have that $M_3(3,F) = <a,b,c,u,v,z>$, where

$$a := \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}, \; b := (I_2 - \varepsilon)^{-1} \begin{bmatrix} I_2 & \varepsilon & I_2 \\ \varepsilon & I_2 & I_2 \\ \varepsilon & \varepsilon & \varepsilon^2 \end{bmatrix}, \; c := (I_2 - \varepsilon)^{-1} \begin{bmatrix} I_2 & \varepsilon & \varepsilon \\ \varepsilon & I_2 & \varepsilon \\ I_2 & I_2 & \varepsilon^2 \end{bmatrix}, \; u := \begin{bmatrix} 0 & 0 & I_2 \\ I_2 & 0 & 0 \\ 0 & I_2 & 0 \end{bmatrix},$$

$$v := \begin{bmatrix} I_2 & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon^2 \end{bmatrix} \quad and \quad z := \begin{bmatrix} \bar{z} & 0 & 0 \\ 0 & \bar{z} & 0 \\ 0 & 0 & \bar{z} \end{bmatrix}.$$

It then follows that $M_{11}(6,F) = <t,a,b,c,u,v,z>$, where $t := \begin{bmatrix} \bar{t} & 0 & 0 \\ 0 & \bar{t} & 0 \\ 0 & 0 & \bar{t} \end{bmatrix}.$

The action of t on $M_3(3,E)$ is given in the proof of theorem 5.2.1.

If $p^k \equiv 1 (\mathrm{mod} 3)$, then t acts trivially on a,b,c,u and v; Otherwise we have $a^t = a^2, b^t = c^3, c^t = bc^2, u^t = u \quad and \quad v^t = v.$

In the case when $p^k = 2$, we have $\lambda = \mu = 1$ and so

$$\bar{t} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \; \bar{z} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad and \;\; \varepsilon = \bar{z}.$$

Furtheremore, $(I_2 - \varepsilon)^{-1} = \varepsilon$. We now have a polycyclic generating sequence for $M_{11}(6,2)$.

3.2-The irreducible subgroups of $M_3(6,2)$.

Recall that $M_3(6,2) = M_2(2,2)\ \ wr\ \ S_3 = GL(2,2)\ \ wr\ \ S_3$. Therefore M_3 has order $2^4 3^4$. Applying lattice and then GETIRR shows that there are 19 M_3-conjugacy classes of irreducible subgroups of M_3.Let irred be a set of representatives of these clases.The orders of the groups in irred are as follows.One each of orders 9 ' 18 ' 81 ' 108 and 12 ' 6 ' two each of orders 27 and 324 ' three each of orders 162 and 648 and four of orders 54.The two groups of order 27 are not isomorphic and not $GL(6,2)$-conjugate.The two groups of order 324 have derived groups of different orders.The three groups of order (162 have pairwise non-isomorphic derived groups.The three groups of order 648 are pairwise non-isomorphic because they differ in the orders of their derived groups and their numbers of conjugacy classes of elements.This leaves the four groups of order 54.Denote by $E_{27}$the extraspecial group of order 27 and exponent 3.By calculating the derived groups of the four groups of order 54 ' we see that one has C_9.One has $E_{27}$and two have $C_3 \times C_3$.

These last two have E_{27} for Fitting subgroup.We know from the theory that leads to the construction of $M_{11}(6,2)$ that $N_{GL(6,2)}(E_{27})/E_{27} \cong M_{11}/Fit(M_{11}) \cong GL(2,3)$.

Since $GL(2,3)$contains a unique conjugacy class of non-central involutions ' we conclude that the last two groups in irred are conjugate in $GL(6,2)$.

Thus there are exactly **18** $GL(6,2)$-conjugacy classes of irreducible soluble subgroups whose guardian is M_3.

3.3-The irreducible subgroups of $M_6(6,2)$.

Recall that $M_6(6,2) = M_2(3,2)\ \ wr\ \ S_2 = (C_7 \succ C_3)\ \ wr\ \ S_2$. Therefore M_6 has order **$2.3^2.7^2$**.Applying lattice and then GETIRR shows that there are six M_6-conjucacy classes of irreducible subgroups of M_6.
Let irred be a set of representatives of these classes.The orders of the groups in irred are as follows:one each of orders 14 ' 42 ' 98 and 882 ' and two of order 294.The two groups of order 294 have derived groups of different orders.

So these six groups are pairwise non-conjugate in $GL(6,2)$. Furtheremore ' none canbe conjugate to a subgroup of M_3 because M_3 dosenot contain any non-trivial **7**-elements.Thus there are exactly six $GL(6,2)$-conjugacy classes of irreducible soluble subgroups whose guardian is M_6.

3.4-The irreducible subgroups of $M_8(6,2)$.

Recall that $M_8(6,2) = C_{63} \times C_6$. Therefore M_8 has order $2.3^2.7$.Applaying lattic and then GETIRR shows that there are 14 M_8-conjugacy classes of irreducible subgroups of M_8 .Let irred be a set of representatives of these classes.

Five members of irred are M_6-conjugate to a group of order 54 in irred. This group has for its Fitting subgroup an extraspecial group of order 27 and exponent 9.This kind of Fitting subgroup is not allowed for primitive groups and so these five groups are impritive. Each of the remaining nine groups in irred contains a cyclic group of order 21 ' which is primitive because it dose not apper on our lists for M_3 *and* M_6.Thus these nine are primitive.The order of these groups are as follows:one each of orders 21 ' 42 ' 189 and 378 ' two of order 126 and three of order 63.The two groups of order 126 have derived groups of different orders.ISOTEST shows that the three groups of order 63 are pairwise non-isomorphic.Thus there are exactly nine $GL(6,2)$-conjugacy classes of primitive soluble subgroups whose guardian is M_8.

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